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Application of Fully Stressed Design Procedures to Wing and Empennage Structures

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A review of two automated fully stressed design procedures for wing and empennage structures is given. One of the methods is based upon average stresses that occur in each structural element. The other, called the nodal stress method, uses internal nodal forces in a smoothing procedure, and yields results that are somewhat more acceptable for design. The nodal stress method is adapted for application to winglike structures with fibrous composite skins. The strength criterion employed is a simplified one, suitable for use in preliminary design. Based upon this criterion, the numbers of layers of fibers in several predetermined directions are tailored for minimum weight. The method is demonstrated by applying it to the stabilizer of a supersonic aircraft, the covers of which are designed in boron fiber/epoxy composite.

Discussion

Background

DEVELOPMENT of automated methods for the optimization of airframe structures could be said to be progressing currently along two rather different paths. On the one hand, one has the fundamental approach, in which the weight of the structure is treated as a mathematical function to be minimized, subject to various constraints. These may include limitations on strength, displacements, vibrational frequencies, and even flutter speed.[1-4] Rather sophisticated techniques must necessarily be employed in this work.

On the other hand, one has the pragmatic approach, in which traditional design procedures are more nearly followed. Here, the structure is first sized for strength, based upon the fully stressed design concept. This implies an idealized structure for which the stresses in all the elements are at prescribed maximum limits under at least one loading condition, or the elements are at minimum areas, thicknesses, etc., as dictated by various other practical considerations. Engineers have long believed that such designs are either at or close to minimum weight, provided of course they are critical for strength. Thereafter, the structure is checked for flutter and/or possibly other dynamic effects as well. If it is found to be deficient, estimates of the increased stiffness required are made, and the structure is modified accordingly. Venkayya et al.[5] have essentially proposed an efficient scheme which can be used for modifying structures to increase their stiffness while continuing to meet strength requirements.

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The aerospace industry is naturally interested in both approaches: the fundamental one because of its potentially greater reductions in airframe weight in the long run, but the pragmatic one too, because of its more immediate applicability. The current paper indicates how the fully stressed design procedure may be applied to the preliminary design of airframe components, and also to those entering the detail design phase.

Average Stress Method

Perhaps the most common approach to idealizing a wing or empennage structure in a finite element analysis is by means of bars, in combination with quadrilateral membrane elements of some sort. The bars are usually uniformly stressed, while some kind of average biaxial stress state can be identified for each of the quadrilaterals.

It is quite natural to attempt to optimize such structures based upon these stresses. Thus, in the case of a fully stressed design, one first performs a redundant analysis for the various design conditions. The highest stress state occurring in each of the elements is then sought out, and based upon it and a suitable design criterion, each element is independently resized. The resized structure is then reanalyzed. The analysis and redesign procedure is automatically repeated until the process converges to a design in which every element is either 1) stressed to its maximum for the given failure criterion under at least one load condition or 2) is at some prescribed minimum size.

As recounted in a previous paper,[6] it has been Grumman's experience that the average stress approach sometimes yields discontinuities in material distributions that are sufficiently erratic to make them undesirable for use in design. This has led to the development of the nodal stress method, which inherently contains a beneficial smoothing procedure. As previously presented, the nodal stress method was applicable only to a structure made of a single, homogeneous material. The following review includes changes necessary to accommodate orthotropic construction, and use of more than one material.

Nodal Stress Method

Airframe redundant structure analysis at Grumman is currently carried out by use of the displacement method. Nevertheless, in the case of a wing or empennage structure, the results of such an analysis are converted into a form consistent with a force method idealization, for use by the stress analyst.[6] The internal loads thus are presented to the analyst in the form of unequal end loads and shear flows acting upon equivalent bars, and shear flows acting upon equivalent shear panels. Panels actually loaded in shear only, such as spar and rib webs, are left unchanged.

The nodal stress method is a resizing procedure based upon stresses derived from this alternate form for representing the internal loads. Otherwise, it is the same as the average stress method.

The first step in the procedure consists of obtaining a set of stress resultants in each corner of every panel. These are found in the following way. The equivalent bars represent 1) the actual stringers and spar or rib caps and 2) the proper amount of cover material, also carrying direct stress, on both sides of the equivalent bar's center line. The part of the equivalent bar end load carried in the cover material is determined, based upon the relative extensional rigidities of the appropriate elements contributing to the given bar. This load is then divided by the total width of cover associated with the bar, yielding the corresponding direct stress resultant. (Note that if there is no spar, rib or stringer material, the direct stress resultant is the entire equivalent bar load divided by the total width of cover.) The accompanying shear stress resultant is taken to be the shear flow acting upon the companion equivalent shear panel. This is discussed in greater detail in Appendix A.

If the spars and ribs of the structure being investigated are approximately orthogonal, the stress resultants so obtained are sufficient to determine a revised thickness at each corner of every panel. Of course a suitable design criterion is needed, as well. On the other hand, if sweep effects are substantial, the stress resultants previously discussed must first be transformed into an orthogonal set.[6] In either event, after the resizing takes place, the four corner thicknesses for each panel are averaged, to provide new values for use in the next analysis cycle. In the case where the cover material is orthotropic, the "thickness" will naturally require special interpretation.

Resizing the spar caps, rib caps and stiffeners presents special problems. Experience to date indicates that if these members are treated as independent elements, as in the average stress method, the results can be very erratic. More realistically, in an actual design the areas of such axial members may be determined by any one of a number of requirements. At one extreme, a spar or rib cap may be little more than a shear connection between the spar or rib web and the cover, in which case the area is minimal. At the other extreme, one has stiffeners which are integral with cover material, and whose area should maintain a fixed relationship with the accompanying skin area for optimum weight. In view of these variations and uncertainties, Grumman has generally found it expedient to hold axial member areas fixed during a fully stressed design calculation. If the results indicate that it would be profitable to do so, one can always assign the areas new values, and the fully stressed design calculation can then be rerun.

Strength Criteria and Element Optimization

The actual strength criteria employed in an airframe design play a vital role in any optimization procedure. In some instances, such as a metallic lower skin of a wing loaded primarily in tension, and with some shear as well, a simple criterion such as one based upon the Hencky von Mises stress may be appropriate. The equivalent shear web of a spar or rib, traditionally used to idealize honeycomb construction, is another example. Here a maximum shear stress allowable would most likely be chosen.

In other situations, more sophisticated criteria may be needed. In the case of a metallic skin and stringer type of wing cover construction, loaded in compression, the allowable strength would depend upon the load levels to which it is subjected. One approach would then be to employ curves in the fully stressed design procedure, which relate allowable stresses to appropriate structural indices. These curves would be based on optimum panel proportions. Since these proportions however, can seldom be exactly implemented in practical designs, some weight penalties might have to be accepted. The idealized, optimized structure nevertheless does provide useful guide lines to the designer.

A further opportunity exists to use optimized elements when the structure includes fibrous composite construction. Here, one can actually tailor the numbers of layers of fibers, and their orientation, in such a way as to achieve minimum weight.

The strength criteria that are just now evolving for the boron fiber/epoxy matrix composites are in general rather complex. One of the many complications, as compared to more conventional skin materials, is the low interlaminar shear rigidity of the composite construction. This can lead to a number of additional compressive instability problems which are not present in homogeneous skin structures. Nevertheless, there are situations where a simple strength criterion can be realistically employed. One of these exists for the supersonic aircraft stabilizer which is considered in this paper. In this case, the skin is supported laterally by a full depth aluminum honeycomb core, and at least for preliminary design purposes, can be regarded as not subject to compressive instability. Accordingly, a simple ultimate strength theory is employed, and is reviewed in Appendix B.

The optimization of a laminate under a given set of stress resultants can also be rather complex. However, there are practical manufacturing constraints that ease this problem. Most importantly, the fibers are limited in the number of orientations that they may assume. For a wing or stabilizer, Grumman designers favor a single spanwise direction as the longitudinal fiber direction. Fibers are then laid at 90° to this direction, and also equal layers in the $\pm 45°$ directions. The latter condition has been found necessary to prevent warping of the composite skin during the curing process. Any deviation from this single set of fiber axes will of course require some sort of splice at the interface between the adjacent regions. A review of the optimization procedure for obtaining the number of layers in each fiber direction is given in Appendix C. In adapting the nodal stress method to structures with boron composite skins, an additional step is necessary. The stress resultants at each corner of every panel must first be rotated into the spanwise fiber direction. An optimized laminate can then be selected for each corner. The numbers of fibers in each of the respective directions are then averaged. These averages for each panel are now rounded up to the next higher integers, to define the structure to be used in the next analysis cycle.

Example

The example structure which will be considered is shown in Fig. 1. It is a stabilizer suitable for a supersonic aircraft, and is of conventional design except for the skins, which are of boron/epoxy composite. The entire surface pivots about a shaft which projects from the fuselage, with actuation accomplished by means of a control horn.

The leading edge, trailing edge and tip assemblies are of full depth aluminum honeycomb, and have aluminum skins. The box beam substructure assembly also has a full depth aluminum honeycomb core, except in the pivot region, where the intercostal beams and the root and intermediate ribs are of titanium. The box beam covers are of boron/epoxy composite, as mentioned previously. A feature of these covers is that in the pivot region they splice into titanium skins, which in turn fasten to the titanium portion of the substructure.

The zero degree boron fibers are parallel to the 50% chord line, except for those inboard of the intermediate rib and forward of the titanium panel. In this region the property axes have been rotated clockwise relative to the rest in order to orient the fibers more normally to the root rib. Thus, a splice exists between the two fiber orientations along the forward portion of the intermediate rib.

The idealized structure selected for the finite element analysis is shown in Fig. 2. As can be seen, it encompasses the box beam, but excludes the leading and trailing edge and tip assemblies. The structure and loading are symmetrical about the mean chord plane, so only the upper cover is shown. The honeycomb core has been compressed into discrete, spar-like and rib-like webs of equivalent shear stiffness. The covers are represented by quadrilateral membrane elements, each constructed from four constant strain triangles, and with appropriate orthotropic stiffness characteristics. There are bar elements to represent the concentrations of axial load carrying material along the front, rear, and intercostal beams and the three closure ribs. Support point loads are distributed to adjacent nodes through additional bar elements. The total number of structural elements employed in the analysis is approximately 1000 while there are approximately 1100 degrees of freedom.

The design conditions consist of four positive loading cases and four equal and opposite negative loading cases. The corresponding centers of pressure vary from a 25% to a 50% chordwise position, and from a 38% to a 50% position spanwise. These applied loads are introduced into the idealized structure as concentrated forces at each of the node points.

First Design—No Minimum Sizes

The selection of minimum sizes for a practical design is always a problem, and is much easier to do after one has a reasonable approximation to the structure's internal load distribution. This information can be readily obtained by first running the structure through several redesigns with no minimum gages specified. The results will be interesting in their own right, as well.

The example structure is accordingly first run through five redesign cycles wih no minimum sizes specified. However, a minimum of one layer of boron fibers in each of the four predetermined directions is maintained. (This is termed a 1, 1, 2 layup, the 2 representing the total number of layers in the $+45°$ and $-45°$ directions.) Also, the spar and rib cap areas are set equal to zero, the cover material alone being capable of carrying the necessary loads.

The strength criterion used for the equivalent discrete aluminum shear webs is a shear stress of 20 ksi. The corresponding value for the titanium shear webs is 85 ksi. In the case of the titanium plate material used in the covers, the strength criterion is the Hencky von Mises yield stress, set equal to 117 ksi. These values for the titanium may be high in some regions of the structure, because they do not allow for interaction between the direct stress in the covers and the

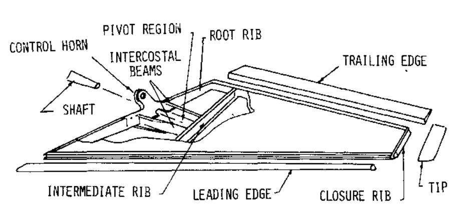

Fig. 1 Stabilizer concept.

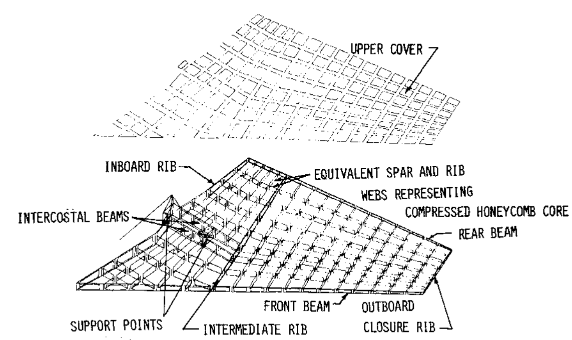

Fig. 2 Regions of constant numbers of fibers after 5 resizings—first design.

shear stress in the webs. However, they are adequate for preliminary design. As for the boron covers, the strength criterion and optimization procedure outlined in Appendices B and C have been followed. Both use layer strengths and matrix effectiveness factor B from Ref. 7, as follows: $F_{1tu} = F_{1cu} = 149$ ksi at 350°F; $F_{xy}' = 1.3$ ksi at 350°F; $F_{2cu}' = 0$; $B = 0.9$. Also, from Ref. 7, the longitudinal and shear moduli of elasticity of a layer are taken to be $E_{11} = 28.8 \times 10^6$ psi at 350°F; $G_{12} = 0.22 \times 10^6$ psi at 350°F. The transverse modulus of a layer is assumed to make a negligible contribution to laminate stiffness in a practical layup, and is set equal to zero, as are the Poisson ratio terms ν_{12} and ν_{21}.

The initial skin gages are chosen for convenience as follows: titanium shear webs—0.1 in.; aluminum shear webs—0.1 in.; titanium in cover—varies—0.125 to 0.2 in. Similarly, the initial cover laminate configuration is six layers of fibers in each of the four directions.

The spar and rib web results from the five redesigns that were carried out are not given here. They have been examined however, and show a reasonable distribution of material, which is in general consistent with the applied loads to be supported. As for the covers, the results of the fifth redesign are shown on Fig. 3. Here, contours are given of numbers of layers required in the four directions. A study of the intermediate cycles indicates that most of the panels will be changed little, if at all, by additional cycling.

After the second redesign the total weight of the idealized structure stabilizes at 108 lb. Of this total, 25% is in the titanium and aluminum substructure, 19% is in the titanium cover splice plate, while 56% is in the boron composite.

Several interesting conclusions may be drawn from Fig. 3. As expected, the span-wise fibers tend to be reasonably symmetric in distribution about the 50% chord line, and tend to peak roughly in the vicinity of the outboard stabilizer attachment point. However, a large part of the load is taken out of the boron and into the titanium splice plate in shear along the aft edge of the splice plate, rather than in direct stress along the splice plate's outer edge. The contours of 90° fibers and $\pm 45°$ fibers are consistent with this. This and other shortcomings are alleviated by considerations discussed in the following paragraphs.

Revised Design Incorporating Minimum Element Sizes

The preceding results are actually unrealistic in a number of ways. For one thing, the titanium splice plate is subject to compressive instability in the local region over the shaft from the fuselage, where it is unsupported by honeycomb core material. In this area, the Hencky von Mises strength criterion used is inapplicable. Proper thicknesses based upon buckling can be selected for the five panels involved, and introduced as minimum gages. The splice plate thickness will also have to be increased in order to develop sufficient bearing strength for the fasteners to the substructure. Furthermore, more material will have to be provided around its periphery in order to make the splice with the boron laminate.

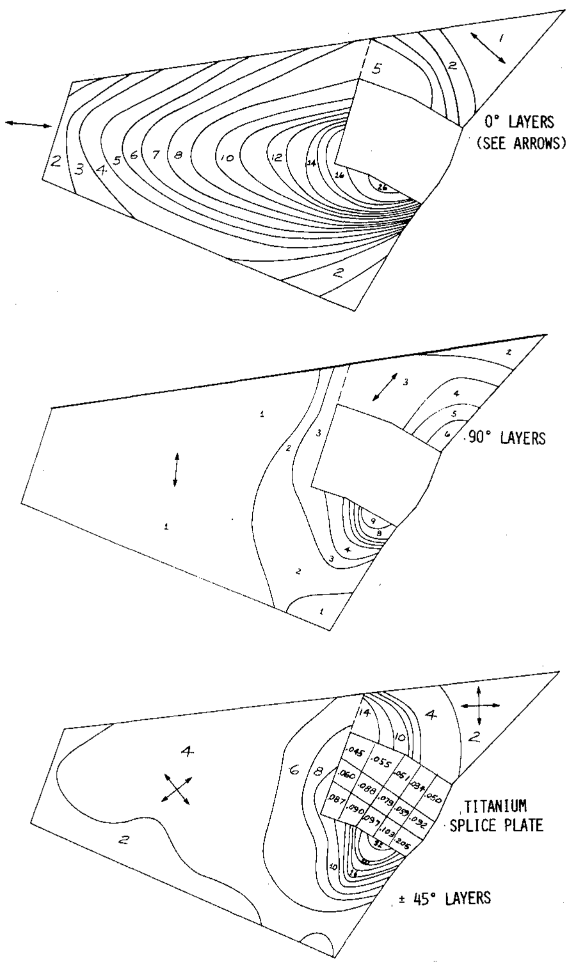

Fig. 3 Structural idealization.

As for the substructure, the honeycomb material will have to conform to whatever is commercially available. The titanium shear web material will have to be compatible with manufacturing minimum gages. And finally, one must provide spar and rib cap material sufficient to transfer shear loads, through fasteners, from these members to the covers.

The preceding comments have neglected the possibility that aeroelastic requirements might also influence the design. In fact, simple calculations show that in a typical application, additional torsional rigidity might well be required.

These considerations have led to the selection of reasonable minimum sizes for all parts of the structure. Included is an increase in the minimum number of $\pm 45°$ boron fibers from 2 to 4, to provide additional torsional rigidity.

Using the minimum sizes discussed, the structure has again been run through five redesign cycles. The results for the covers are presented in Fig. 4.

A study of the contours presented indicates that in general the loads are still carried in the same manner as for the earlier design. However, the considerably stiffer titanium cover plate causes more of the load to be transferred into it by direct stress along its outer edge, and less by shear along the aft edge.

From the practical point of view, it would not be desirable to adhere rigidly to the fiber distributions in these figures. Instead, one would logically use fewer layers than the peak values shown in some regions, and more layers to compensate, in adjacent regions.

As an indication of the dramatic influence that practical minimum sizes may have, the total weight for the redesigned structure stabilizes at 233 lb (against the earlier weight of 108 lb), of which now 35% is in the titanium and aluminum substructure, 20% in the titanium cover splice plate, and 45% is in the boron composite and the titanium spar and rib caps.

Concluding Remarks

It should be reiterated that strength criteria for airframe structures can be fairly complex, as in the case of the boron composite discussed in this paper. Opportunities exist to optimize the elements of such structures as well, and this adds further complications. An automated program of the type described herein can tremendously speed up the work entailed, and permit the designer to investigate changes in design which he might otherwise not have time to do. A simple change in orientation of the boron fibers is a possible example.

Fully stressed designs are clearly not the whole answer to the designer's problems. Obviously, one would prefer further automation, both to speed up the redesign for flutter and other stiffness related effects, and to be able to modify the results if desired, so that they are true optimums. Such methods may soon be available. Reference 4 offers a good deal of hope along these lines.

What is not so obvious perhaps is that a theoretically optimum structural design may not bear much resemblance to the final article when it is actually built. In the case of the boron stabilizer, even if only minor changes were needed for flutter requirements, the manufacturing, fabrication and joining requirements similar to those mentioned might still dictate substantial changes. In spite of such essential compromises, the automated optimum design procedures currently on the horizon will be a big help to the designer.

Appendix A: Calculation of Stress Resultants in a Panel Corner

The nodal stress method requires that the results of a displacement type analysis first be converted into force method form, where the structure is idealized as an assemblage of bars and shear panels. Figure 5 shows such an idealization where

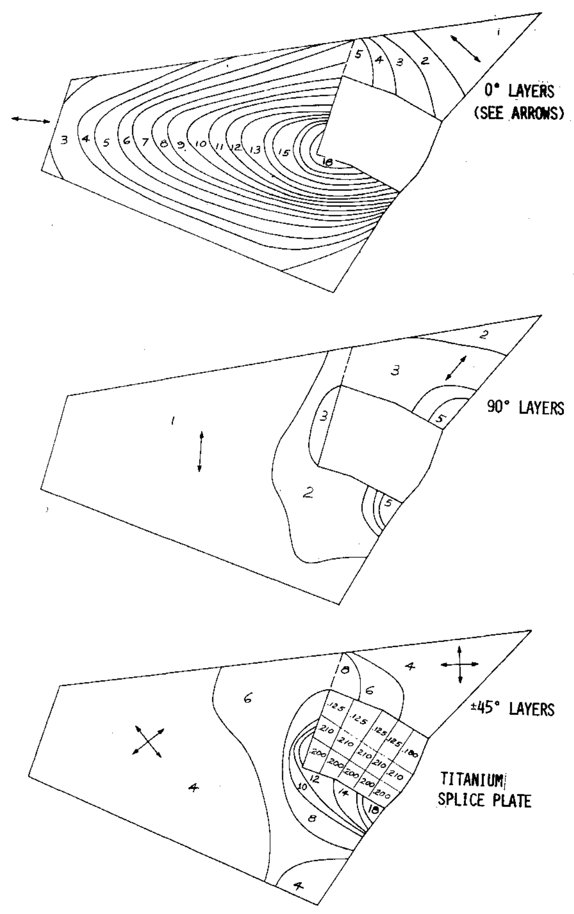

Fig. 4 Regions of constant numbers of fibers after 5 resizings—revised design.

one set of bar end loads, P_s in the sparwise direction and P_r in the ribwise direction, is obtained by summing the corner forces of the elements framing into the node. The shear flows, q_{ss} in the sparwise direction, and q_{rr} in the ribwise direction, are the differences in the panel corner forces divided by the lengths of the appropriate sides.

To determine the stress resultants at the corner of panel 1 (Fig. 5), it is necessary to distribute P_s and P_r between the bars and panels. A reasonable way of doing this is to require the panels to carry a fraction of the nodal force depending upon: 1) the relative amounts of direct stress carrying area in the bars and panels measured normal to the spar or rib axis; 2) the relative elastic moduli (stiffnesses) of the bars and panels in the sparwise and ribwise directions. The stress-strain law for an orthotropic element relative to its property axes is:

$$\begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{12} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & 0 \\ A_{21} & A_{22} & 0 \\ 0 & 0 & A_{33} \end{bmatrix} \begin{Bmatrix} \epsilon_{11} \\ \epsilon_{22} \\ \epsilon_{12} \end{Bmatrix} \tag{A1}$$

where σ_{11}, σ_{22}, σ_{12} are the longitudinal, transverse, and shear stresses; ϵ_{11}, ϵ_{22}, ϵ_{12} are the longitudinal, transverse, and shear strains; A_{ij} $(i, j = 1, 2, 3)$ are functions peculiar to the particular orthotropic material in question. This stress-strain relationship is also used to develop the stiffness matrix for a boron composite membrane element in the redundant analysis.

Let A_{ij}^* represent an element of the elastic property matrix transformed to the local axes of the panel. For the case shown in Fig. 5, side i-j of panel 1 (sparwise direction) corresponds to the longitudinal axis. The following expressions are used to determine the sparwise and ribwise stress resultants, N_s and N_r.

$$N_s = \frac{P_s}{l_{s1} + l_{s2}} \left[\frac{t_1 l_{s1}(A_{11}^*)_1 + t_2 l_{s2}(A_{11}^*)_2}{t_1 l_{s1}(A_{11}^*)_1 + t_2 l_{s2}(A_{11}^*)_2 + \alpha_s E_s} \right] \tag{A2}$$

$$N_r = \frac{P_r}{l_{r1} + l_{r3}} \left[\frac{t_1 l_{r1}(A_{22}^*) + t_3 l_{r3}(A_{22}^*)_3}{t_1 l_{r1}(A_{22}^*)_1 + t_3 l_{r3}(A_{22}^*)_3 + \alpha_r E_r} \right] \tag{A3}$$

where t_i = thickness of panel i; l_{si}, l_{ri} = half-length of panel i measured normal to spar or rib axis (Fig. 5); a_s, a_r = area of bar representing stringer, spar or rib cap; E_s, E_r = modulus of elasticity of bar in spar or rib direction. The use of A_{ij}^* terms in these expressions involves the assumption that the panel is restrained in the transverse direction when considering extensional stiffness in the longitudinal direction. So, for example, for a titanium panel, $A_{11} = A_{22} = A_{11}^* = A_{22}^* = E/1 - \nu^2$, a greater effective modulus than for a titanium bar. Also in the absence of spar or rib cap material, it can be seen that the bracketed previous terms reduce to unity. With or without spar cap material, the method yields a single value for the stress resultant N_s acting in the covers on both sides of the equivalent spar. A similar statement holds for N_r. Thus, an expedient method is now included for handling the situation of dissimilar materials along a shear splice. In this respect, the nodal stress method details reviewed here are more general than those given previously.[6]

Finally, the shear stress resultant N_{rs} is obtained by

$$N_{rs} = q_{ss} \text{ or } q_{rr} \tag{A4}$$

whichever is larger. Once these stress resultants are determined, transformation to a local orthogonal system followed by a transformation to the property axes system is necessary before resizing of the corner can take place.

Appendix B: Preliminary Design Allowable Strength and Stiffness

The failure criterion that is used herein for multidirectional boron composite laminates is a modified form of the Hill-von Mises distortional energy criterion. It describes a failure envelope for a 0°, 90°, ±45° laminate subjected to a biaxial state of stress and is written

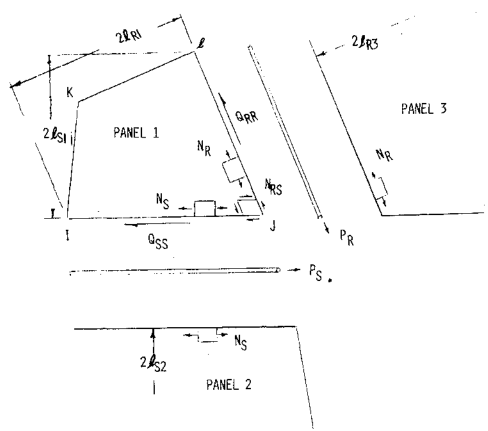

Fig. 5 Stress resultants at a typical spar-rib intersection.

$$F_1 = (N_x/F_x)^2 + (N_y/F_y)^2 - N_xN_y/F_xF_y + (N_{xy}/F_{xy})^2 \leq 1 \tag{A5}$$

$$F_2 = (N_x/F_x)^2 \leq 1 \tag{A6}$$

$$F_3 = (N_y/F_y)^2 \leq 1 \tag{A7}$$

where N_x, N_y, and N_{xy} are the longitudinal, transverse and shear stress resultants applied to the laminate (lb/in.) and F_x, F_y, and F_{xy} are the uniaxially loaded allowable longitudinal, transverse and shear strengths of the laminate (lb/in.).

Preliminary design expressions for F_x, F_y, and F_{xy} have been used for this paper[7] and it should be kept in mind that these expressions are currently being modified to correlate with more recent test data. The laminate allowables are functions of l, m, and n, the numbers of layers in the 0°, 90° and ±45° directions, respectively, and the unidirectional properties of a layer. They are

$$F_x = F_{1cu}t\left[l + \frac{n}{4} - \frac{(n/4)^2}{m + (n/4)}\right] + mtF_{2cu}' \quad \text{for } N_x < 0 \tag{A8a}$$

$$F_x = BF_{1tu}t\left[l + \frac{n}{4} - \frac{(n/4)^2}{m + (n/4)}\right] \quad \text{for } N_x \geq 0 \tag{A8b}$$

$$F_y = F_{1cu}t\left[m + \frac{n}{4} - \frac{(n/4)^2}{l + (n/4)}\right] + ltF_{2cu}' \quad \text{for } N_y < 0 \tag{A9a}$$

$$F_y = BF_{1tu}t\left[m + (n/4) - \frac{(n/4)^2}{l + (n/4)}\right] \quad \text{for } N_y \geq 0 \tag{A9b}$$

$$F_{xy} = ntF_{xy}^N + (l + m)tF_{xy}' \tag{A10}$$

where F_{xy}^N, the allowable strength in shear of the ±45° layers (psi), is

$$F_{xy}^N = \frac{F_{1tu}}{2[1 + (F_{1tu}/F_{1cu}) + (F_{1tu}/F_{1cu})^2]^{1/2}} \tag{A11}$$

In addition F_{1tu}, F_{1cu} = layer longitudinal tensile and compressive strengths (psi); F_{2cu}' = transverse compressive stress per layer at longitudinal failing strain (psi); F_{xy}' = shear stress in a 0° (or 90°) layer at failing shear strain of the ±45° layers; B = the matrix effectiveness in transferring load from broken to unbroken filaments; t = 0.0051 in. (thickness of a layer).

Optimization of a boron composite laminate consists of finding, for a given set of loading conditions, the minimum value of the sum of l, m, and n which satisfies the Hill-von

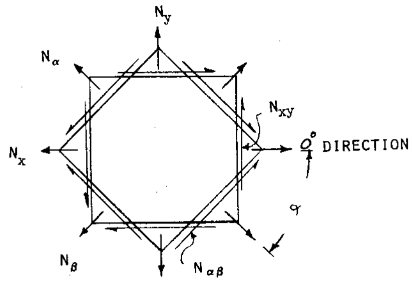

Fig. 6 Typically loaded laminate.

Mises failure criterion. Appendix C is a description of a technique for accomplishing this.

The laminate stiffness characteristics are based upon the standard approach reviewed in Ref. 7. The assumption is made herein that, of the layer properties, only the longitudinal modulus E_{11} and the shear modulus G_{12} need be taken different from zero. The resulting coefficients entering the defining Eq. (A1) are

$$A_{11} = [(l + n/4)/(l + m + n)]E_{11} + [n/(l + m + n)]G_{12}$$
$$A_{12} = [(n/4)/(l + m + n)]E_{11} - [n/(l + m + n)]G_{12} \quad \text{(A12)}$$
$$A_{22} = [(m + n/4)/(l + m + n)]E_{11} + [n/(l + m + n)]G_{12}$$
$$A_{33} = [(n/4)/(l + m + n)]E_{11} + [(l + m)/(l + m + n)]G_{12}$$

Appendix C: Optimization of Boron Composite Laminates

A typical state of stress in a boron composite element is shown in Fig. 6 where the applied stress resultants are oriented at an angle α to the 0° property axis of the laminate. In Fig. 6, N_α, N_β, $N_{\alpha\beta}$ are applied longitudinal, transverse and shear stress resultants. N_x, N_y, N_{xy} are the stress resultants after rotation into the property axes of the element.

Optimization of the laminate may be thought of as the mathematical minimization problem. Find the minimum of (l, m, n) with the inequality constraints $F_1 \leq 1, F_2 \leq 1, F_3 \leq 1$ (from Appendix B), $F_3 \leq 1$. Optimization of the laminate begins by choosing an initial layup (l,m,n) to carry the loads. This initial layup is chosen by selecting the maximum absolute values of N_x, N_y, and N_{xy} from among all the loading conditions. The initial number of n layers is then determined by assuming the n layers carry all the N_{xy} stress. This results in Eq. (A10) being reduced to $N_{xy} = ntF_{xy}{}^N$ which can be solved for n directly. This value of n is then rounded down to the first smaller even integer. Using this value of n, Eqs. (A8) and (A9) can be substituted into Eqs. (A6) and (A7) resulting in two simultaneous equations that may be solved for l and m. These values of l and m are rounded down to the next lower integers. This starting layup may be unconservative because of the rounding down and/or because of the neglect of the interaction of N_x, N_y, and N_{xy} in Eq. (A5).

If this starting layup does not result in a point satisfying the constraint equations the incremental change of l, m, or n that will result in the greatest decrease in the function F per unit increase in the laminate thickness is determined. This is done by computing the value of the function F at $(l + 2, m, n)$, $(l, m + 2, n)$, $(l, m, n + 2)$. The change that gives the smallest value of F is then selected as a new layup. This is repeated until a combination of l, m, and n that gives a value of the function F less than one is determined. At this point the values of F at $F(l - 1, m, n)$ and $F(l, m - 1, n)$ are checked to see if one layer in the l or m direction may be subtracted from the laminate. Because of the way the initial starting size is chosen this procedure almost invariably results in a minimum weight laminate.

To investigate the possibility that the local minimum was not the global minimum a plot of the constraint surface in l, m, n space was examined for several different loading conditions. In each case, the constraint surface had only one minimum.

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